Algebra
- with n linear equations, and m < n unknowns, at most m values are needed to solve
  - [linear coefficients] x [unknowns] = [non-homogenous constants]
  - [n x m][m x 1] = [n x 1]

Calculus
- order of n order differential = order of n order system of 1st order differentials
  - n values required for n order differential to solve
  - n values required for n order system of 1st order differentials to solve

Mass Action
- general n order system of 1st order differentials

$$\frac{dx_i}{dt} = \sum_{j=1}^{a_i} b_{ij} \prod_{k=1}^{c_{ij}} x_{d_{ijk}}^{e_{ijk}} = \sum_{j=1}^{a_i} b_{ij} \prod_{k=1}^{\sum_{h=1}^{c_{ij}} e_{ijh} = \sum_{h=1}^{c_{ij}} f_{ijd_{ijh}} = \sum_{h=1}^{g_{ij}} f_{ijh} = l_{ij}} x_{d_{ijk}} = \sum_{j=1}^{a_i} b_{ij} \prod_{k=1}^{g_{ij}} x_k^{f_{ijk}} = \sum_{j=1}^{m} b_{ij} \prod_{k=1}^{n} x_k^{f_{ijk}}$$

  - parameters
    - x is the species vector
    - a is an amount of rates changing x
    - b is a rate constant
    - c is a number of different species involved
    - d is a species involved
    - e is a number of each species involved
    - f is a number of species d involved
    - g is a highest species involved
    - l is a rate order
    - m is the highest number of rates involved
    - n is the number of species
  - general rate equation

$$\sum_{i=1}^{n} r_{ij} = \sum_{l=0}^{o} \sum_{i=1}^{n} r_{lij} \le o$$

$$\frac{dx_i}{dt} = \sum_{j=1}^{m} (p_{ij} - r_{ij}) c_j \prod_{k=1}^{n} x_k^{r_{kj}} = \sum_{l=0}^{o} \sum_{j=1}^{m} (p_{lij} - r_{lij}) c_{lj} \prod_{k=1}^{n} x_k^{r_{lkj}} = \sum_{l=0}^{o} \sum_{j=1}^{m} (p_{lij} - r_{lij}) c_{lj} \prod_{k=1}^{l} x_{h_{jk}}$$

$$\frac{dx}{dt} = \sum_{j=1}^{m} (P_j - R_j) c_j \prod_{k=1}^{n} x_k^{r_{kj}}$$

    - x is the species vector
    - m is the number of reactions
    - p is a stoichiometric coefficient of the product
    - r is a stoichiometric coefficient of the reactant
    - c is a reaction rate constant
    - n is the number of species
    - h is a species involved
    - P is the stoichiometric coefficient matrix for products
    - R is the stoichiometric coefficient matrix for reactants
- simplifications
  - simplified n order system of constant differential:

$$\frac{dx}{dt} = a$$

    - x is a species vector
    - a is a rate vector
    - solution

$$x(t) = at + x(0)$$

    - calibration

$$a = (x(t) - x(0))\frac{1}{t}$$

    - simplified zero order rate equation

$$r_{ij} = 0$$

$$\frac{dx_i}{dt} = \sum_{j=1}^{m} p_{ij} c_j$$

$$\frac{dx}{dt} = Pc = a$$

      - uses rate equation notation
      - solution

$$x(t) = Pct + x(0)$$

      - calibration

$$c = P^{-1}(x(t) - x(0))\frac{1}{t}$$

  - simplified n order system of linear differential:

$$\frac{\partial}{\partial t} x = Ax + b$$

    - x is a species vector
    - A is a linear coefficient matrix
    - b is a non-homogenous constant vector
    - solution

$$x(t) = e^{At}\left(x(0) + A^{-1}b\right) - A^{-1}b$$

    - steady state

$$x(\infty) = -A^{-1}b$$

    - stability

$$Ax = \lambda x$$

$$\lambda < 0$$

    - optimization

$$\frac{dx(\infty)}{db} = -A^{-1} \ne 0$$

$$\frac{dx(\infty)}{dA} = A^{-2}b = 0 = b$$

    - calibration

$$A = -\ln\left(\left(x(0)1^T - x(\infty)1^T\right)\left(X(t) - x(\infty)1^T\right)^{-1}\right)\frac{1}{t} = -\ln\left((X(0) - X(\infty))(X(t) - X(\infty))^{-1}\right)\frac{1}{t}$$

$$b = -Ax(\infty) = \ln\left(\left(x(0)1^T - x(\infty)1^T\right)\left(X(t) - x(\infty)1^T\right)^{-1}\right)\frac{x(\infty)}{t} = \ln\left((X(0) - X(\infty))(X(t) - X(\infty))^{-1}\right)\frac{x(\infty)}{t}$$

      - X(t) is the matrix of measured data
    - simplified 1st order rate equation

$$\sum_{i=1}^{n} r_{ij} = \sum_{l=0}^{1}\sum_{i=1}^{n} r_{lij} \leq 1$$

$$\frac{dx_i}{dt} = \sum_{j=1}^{m} (p_{ij} - r_{ij}) c_j x_{h_j}^{r_{hij}} = \sum_{l=0}^{1}\sum_{j=1}^{m} (p_{lij} - r_{lij}) c_{lj} x_{h_j}^{r_{lhij}} = \sum_{j=1}^{m} (p_{1ij} - r_{1ij}) c_{1j} x_{h_j} + \sum_{j=1}^{m} (p_{0ij} - r_{0ij}) c_{0j} = \sum_{l=0}^{1}\sum_{j=1}^{m} (p_{lij} - r_{lij}) c_{lj} \prod_{k=1}^{l} x_{h_{jk}}$$

$$\frac{dx}{dt} = \sum_{j=1}^{m} (P_j - R_j) c_j x_{h_j}^{r_{hij}}$$

- uses rate equation notation

- simplified n order system of quadratic differential:

$$\frac{dx_i}{dt} = x^T A_i x + b_i' x + c_i$$

- x is the species vector
- A is a quadratic coefficient matrix
- b is a linear coefficient vector
- c is a non-homogenous constant
- simplified 2nd order rate equation

$$\sum_{i=1}^{n} r_{ij} = \sum_{l=0}^{2}\sum_{i=1}^{n} r_{lij} \leq 2$$

$$\frac{dx_i}{dt} = \sum_{j=1}^{m} (p_{ij} - r_{ij}) c_j x_{h_{j1}}^{r_{h_{j1}j}} x_{h_{j2}}^{r_{h_{j2}j}} = \sum_{l=0}^{2}\sum_{j=1}^{m} (p_{lij} - r_{lij}) c_{lj} x_{h_{j1}}^{r_{lh_{j1}j}} x_{h_{j2}}^{r_{lh_{j2}j}} = \sum_{j=1}^{m} (p_{2ij} - r_{2ij}) c_{2j} x_{h_{j1}} x_{h_{j2}} + \sum_{j=1}^{m} (p_{1ij} - r_{1ij}) c_{1j} x_{h_{j1}} + \sum_{j=1}^{m} (p_{0ij} - r_{0ij}) c_{0j} = \sum_{l=0}^{2}\sum_{j=1}^{m} (p_{lij} - r_{lij}) c_{lj} \prod_{k=1}^{l} x_{h_{jk}}$$

$$\frac{dx}{dt} = \sum_{j=1}^{m} (P_j - R_j) c_j x_{h_{j1}}^{r_{h_{j1}j}} x_{h_{j2}}^{r_{h_{j2}j}}$$

- uses rate equation notation